the leading-order boundary-layer solution:

$$\frac{Nu_x}{(Nu_x)_{b\text{-}l}} = \frac{1}{[1-0.603\bar{R}\varepsilon_H^{1/2}+0.459\bar{R}^2\varepsilon_H-0.333\varepsilon_H]}. \quad (30)$$

Figure 2 shows the variation of $Nu_x/(Nu_x)_{b\text{-}l}$ with $Ra_x = 1/(\varepsilon_H)^2$, for various $\bar{R}$, using (30). The $\bar{R} = 0$ curve applies in the absence of an external free stream, when the higher-order natural convection boundary-layer corrections up to $O(\varepsilon_H)$ are taken into account. Variations for $\bar{R} = 0.5$, 1 and 2 are for progressively increasing mixed convection effects. From Fig. 2 it is seen that an increase in $\bar{R}$ results in a higher value of the ratio $Nu_x/(Nu_x)_{b\text{-}l}$. It is clear that even moderate values of the mixed convection parameter can result in a 10–25% increase in the local Nusselt number in the range $100 \leqslant Ra_x < 1000$.

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Ambiguities related to the calculation of radiant heat exchange between a pair of surfaces

W. Q. TAO and E. M. SPARROW*

Department of Power Machinery Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, China

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NOMENCLATURE

- A surface area
- E_b black-body emissive power
- $\mathscr{F}$ Hottel's script-F factor
- F angle factor
- $Q_{1,2}$ radiant heat exchange between surfaces 1 and 2 in Oppenheim's definition, equation (1)
- Q_{1-2} radiant heat exchange between surfaces 1 and 2 in Hottel's definition, equation (4)
- T absolute temperature.

Greek symbol
- ε emissivity.

INTRODUCTION

IN THE ANALYSIS of radiant transfer, the desired end result is the net heat transfer at a given surface due to interchange with all surfaces with which it can interact radiatively. Frequently, as an intermediate step in the analysis, the value of the radiant exchange between a pair of surfaces is determined, and the values are summed for all pairs which involve the surface of interest, thereby yielding its net heat transfer. In various heat transfer textbooks, there appears to be some unwitting confusion about the radiant exchange between surface pairs. The objective of this note is to illuminate those issues.

First, it may be noted that the two most commonly used procedures for the analysis of radiant interchange, the Oppenheim network method [1] and Hottel's script-F method [2], yield different results for the radiant exchange between surface pairs. This fact appears not to have been realized before. For example, in [3–5], the pair-specific radiant exchange as given by the two procedures is improperly indicated as being the same. This may be verified by comparing equations (11.56) and (11.60) in [3], equations (11.2) and (11.23) in [4], and equations (11.57) and (11.67) in [5].

Second, in [6], the formula for the net heat transfer between the surfaces in a two-surface enclosure is mistakenly employed to evaluate the radiant exchange between a pair of surfaces which do not form an enclosure.

PAIR-SPECIFIC RADIANT EXCHANGE

In terms of the radiosity, Oppenheim [1] defined the radiant heat exchange between two surfaces 1 and 2 as

$$Q_{1,2} = \frac{J_1 - J_2}{1/(A_1 F_{12})} \quad (1)$$

where A is the radiative surface area and F is the shape factor. For an enclosure of N surfaces, the radiosities at all of the surfaces $i = 1, 2, \ldots, N$ can be determined by solving the following set of N algebraic equations

$$J_i = \varepsilon_i E_{bi} + (1-\varepsilon_i) \sum_{j=1}^{N} F_{ij} J_j, \quad i = 1, 2, \ldots, M \quad (2)$$

$$J_i = \sum_{j=1}^{N} F_{ij} J_j, \quad i = (M+1), \ldots, N \quad (3)$$

where the temperatures are prescribed at surfaces $i = 1$ through M, while surfaces $(M+1)$ through N are no-flux surfaces.

On the other hand, Hottel [2] defined the radiant heat exchange between surfaces 1 and 2 as

$$Q_{1-2} = A_1 \mathscr{F}_{12}(E_{b1} - E_{b2}) = -A_2 \mathscr{F}_{21}(E_{b2} - E_{b1}) \quad (4)$$

* Advisory Professor.

where $\mathscr{F}_{12}$ and $\mathscr{F}_{21}$ are the so-called script-F factors and E_b is the black-body emissive power. As shown in [7], $\mathscr{F}_{ij}$ is determined by setting $E_{bi} = 1$, $E_b = 0$ for all other surfaces except the no-flux surfaces (which need not be specified), and solving the set of equations (2) and (3) for J_j. Then,

$$\mathscr{F}_{ij} = \frac{A_j\varepsilon_j J_j}{A_i(1-\varepsilon_j)}. \tag{5}$$

The fundamental difference between $Q_{1,2}$ and Q_{1-2} is that the latter is truly the net heat transfer between surfaces 1 and 2, whereas this is not so for the former. In particular, as will be illustrated later, $Q_{1,2}$ is not necessarily zero when $E_{b1} = E_{b2}$. On the other hand, equation (4) shows that $Q_{1-2} = 0$ when $E_{b1} = E_{b2}$.

Suppose that a pair of surfaces 1 and 2 with specified temperatures and emissivities are two among various surfaces which comprise an enclosure, and the radiant heat exchange between them is sought. For simplicity, suppose that the enclosure is completed by

I: black surfaces with a common emissive power;
II: no-flux surfaces (i.e. $Q_{net} = 0$ surfaces)

and the surfaces 1 and 2 are flat or convex. Then, according to the definitions, equations (1) and (4), the corresponding formulas for the radiant heat exchange between surfaces 1 and 2 can be derived as

$$Q^{I}_{1,2} = \langle A_1F_{12}\{(F_{23}/\varepsilon_2+F_{21})E_{b1}-(F_{13}/\varepsilon_1+F_{12}) \times E_{b2}+E_{b3}[(1/\varepsilon_1-1)F_{13}-(1/\varepsilon_2-1)F_{23}]\}\rangle / \langle 1/\varepsilon_1\varepsilon_2-(1/\varepsilon_2-1)(1/\varepsilon_1-1)(A_1/A_2)F_{12}^2\rangle \tag{6}$$

$$Q^{II}_{1,2} = \frac{A_1F_{12}\varepsilon_1\varepsilon_2(1-F_{33})(E_{b1}-E_{b2})}{D} \tag{7}$$

$$Q^{I}_{1-2} = \frac{A_1F_{12}(E_{b1}-E_{b2})}{1/\varepsilon_1\varepsilon_2-(1/\varepsilon_1-1)(1/\varepsilon_2-1)(A_1/A_2)F_{12}^2} \tag{8}$$

$$Q^{II}_{1-2} = [A_1F_{12}(E_{b1}-E_{b2})]/[1/\varepsilon_1-1 +(1/\varepsilon_2-1)(A_1/A_2)+(A_2/A_1+1 -2F_{12})/(A_2/A_1-F_{12}^2)] \tag{9}$$

where

$$D = 1-F_{33}-(1-\varepsilon_1)F_{13}F_{31}-(1-\varepsilon_2)F_{23}F_{32} -(1-\varepsilon_1)(1-\varepsilon_2)F_{12}F_{21}+(1-\varepsilon_1)(1-\varepsilon_2) \times F_{12}F_{21}F_{33}-(1-\varepsilon_1)(1-\varepsilon_2)F_{12}F_{23}F_{31} -(1-\varepsilon_1)(1-\varepsilon_2)F_{13}F_{32}F_{21}. \tag{10}$$

To illustrate the difference between $Q_{1,2}$ and Q_{1-2}, these quantities are evaluated for a simple case: a cubical enclosure (side = 1 m) in which surfaces 1 and 2 are facing each other. The respective temperatures and emissivities of these surfaces are $T_1 = 773$ K, $T_2 = 300$ K, $\varepsilon_1 = 0.8$, and $\varepsilon_2 = 0.6$. The other four surfaces are treated according to the previous specifications of cases I and II: for case I, $E_{b3} = 0$. The results for case I are: $Q^{I}_{1-2} = 1905.3$ W and $Q^{I}_{1,2} = 2936.3$ W, whereas for case II, $Q^{II}_{1-2} = 7658.2$ W and $Q^{II}_{1,2} = 2550.0$ W.

It can be seen that $Q_{1,2}$ and Q_{1-2} are different and that the differences are significant. This numerical example is by no means atypical. Thus, the interchange of $Q_{1,2}$ and Q_{1-2} is, in general, inappropriate. Only for a black-body enclosure or an enclosure composed of two surfaces are they equal.

It will now be demonstrated that $Q^{I}_{1,2} = 0$ does not, in general, fulfill the requirement that $T_1 = T_2$. From equation (6) with $Q^{I}_{1,2} = 0$ (and with $E_{b3} = 0$ for simplicity), there follows

$$E_{b1}/E_{b2} = (\varepsilon_2/\varepsilon_1)(F_{13}+\varepsilon_1F_{12})/(F_{23}+\varepsilon_2F_{21}). \tag{11}$$

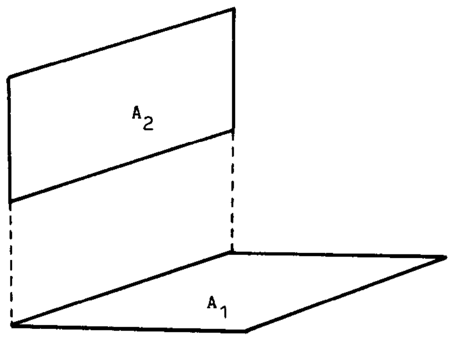

FIG. 1. Configuration to which equation (12) was applied in ref. [6].

For the values and geometry of the foregoing example, $T_1/T_2 = 0.94 \neq 1$. Similarly, it follows from equation (6) that, in general, $Q^{I}_{1,2} \neq 0$ when $T_1 = T_2$.

The foregoing characteristics of $Q^{I}_{1,2}$ affirm that it is not truly a net heat transfer. On the other hand, it follows by direct inspection of equations (8) and (9) that $Q_{1-2} = 0$ is equivalent to $T_1 = T_2$ and vice versa. Consequently, Q_{1-2} possesses the basic character of a net heat transfer.

MISAPPLICATION OF ENCLOSURE-BASED FORMULA

In [6], the net heat transfer between the surfaces of a two-surface enclosure was properly derived as

$$Q_{1-2} = \frac{E_{b1}-E_{b2}}{(1-\varepsilon_1)/A_1\varepsilon_1+1/A_1F_{12}+(1-\varepsilon_2)/A_2\varepsilon_2}. \tag{12}$$

Subsequently, this formula was applied to determine Q_{1-2} for the configuration pictured in Fig. 1, with surface 1 black and surface 2 gray, yielding

$$Q_{1-2} = \frac{E_{b1}-E_{b2}}{1/A_1F_{12}+(1-\varepsilon_2)/A_2\varepsilon_2}. \tag{13}$$

The use of equation (12) in this application is clearly incorrect since surfaces 1 and 2 do not, in themselves, constitute an enclosure. Furthermore, even if the enclosure were completed with surfaces assigned the most elementary thermal conditions, case I with $E_{b3} = 0$ and case II, equation (13) is still invalid.

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